Newtonian Labs

Magneto-Mechanical Harmonic Oscillator

Guide to Experiments

I. Introduction

The goal of this Guide is to give an overview of some of the observations and experiments that can be performed using the *Newtonian Labs* Magneto-Mechanical Harmonic Oscillator (MMHO) instrument. For information about the hardware itself, please see our *Instrument Description* document. And for a more detailed discussion of the underlying physics, please see our *Magneto-Magnetic Harmonic Oscillator Physics* document.

The focus of this experiment is on the physics of the Simple Harmonic Oscillator (SHO). You will see below that the experiments come in different levels, from basic observations to more involved quantitative measurements. With a spectrum of possibilities, the instructor may choose a set of experiments that is appropriate for a given group of students

II. Driven harmonic oscillations

Manual data-taking

The MMHO is especially well suited for exploring the driven harmonic oscillator, as it produces high quality data with remarkable ease. First set the signal generator to produce a sine-wave signal near 40 Hz, and then send this signal to the Drive Coil IN port (see the *Instrument Description* document). The amplitude of the oscillations can be seen directly from the visible laser streak on the graduated ruler. One then measures the oscillator amplitude as a function of drive frequency, for fixed drive amplitude.

One especially simple and intuitive method for measuring the oscillator amplitude is by measuring the length of the laser streak, done by eye on the graduated ruler, and this method yields surprisingly accurate data. The ends of the laser streak are especially bright (because the oscillator angular velocity goes to zero at these points), so measuring a 100-mm laser streak to an accuracy of 1 mm is straightforward. Figure 1 shows example data, taken by hand using this method.

Note that when the oscillator Q is this high, 100 or more, the response curve is fairly well described by a simple Lorentzian function. Fitting this curve to the data yields excellent agreement with theory, as shown in Figure 1.

Note also that the oscillator settles down to its steady-state amplitude in less than ten seconds, so accurate data can be acquired fairly quickly. By the time one records a new frequency setting, the

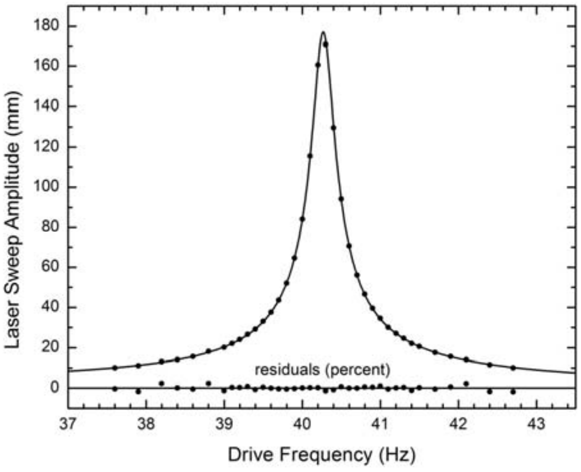

Figure 1. Investigating the driven harmonic oscillator. The upper data points show measurements of the oscillator amplitude as a function of drive frequency, obtained from observing the laser streak on the graduated ruler. The line though the data points is a fit to a Lorentzian function. Residuals are shown in the lower data points.

oscillator has settled into steady-state oscillations, so the amplitude can be measured. With such a rapid relaxation time, the entire driven oscillator response curve can be mapped out by hand in a reasonably short time. Plotting the data then yields quite a satisfying result.

Manual data-taking with the oscilloscope

If using a graduated ruler for data taking does not appeal, one can use a digital oscilloscope to acquire amplitude measurements manually. One simply sends the photodiode signal to the oscilloscope and uses the measure feature to determine the amplitude of the oscillation signal, again writing the numbers down by hand. A digital multimeter would work in this capacity also, measuring the AC voltage. Because the photodiode signal has a higher dynamic range than the laser sweep, using this diagnostic allows measurements of a broader frequency range.

Automated data-taking

If manual data-taking is altogether too old-fashioned for your tastes, even better data can be obtained using automated software, such as LabVIEW (note this software is not included in the MMHO package). The photodiode signal provides a voltage that is proportional to $\theta(t)$, the oscillator angular position as a function of time, and from this signal the oscillation amplitude can be extracted. Once the software is set up, the amplitude can be measured as the drive frequency is changed, and again the data-taking is fast, taking as little as ten seconds per data point.

Figure 2 shows data acquired using this automated method. With a greater number of data points acquired over a larger frequency range, the simple Lorentzian approximation is no longer sufficient, so one must instead use the full SHO response function. Again the basic SHO theory yields an excellent fit to the data, in this case

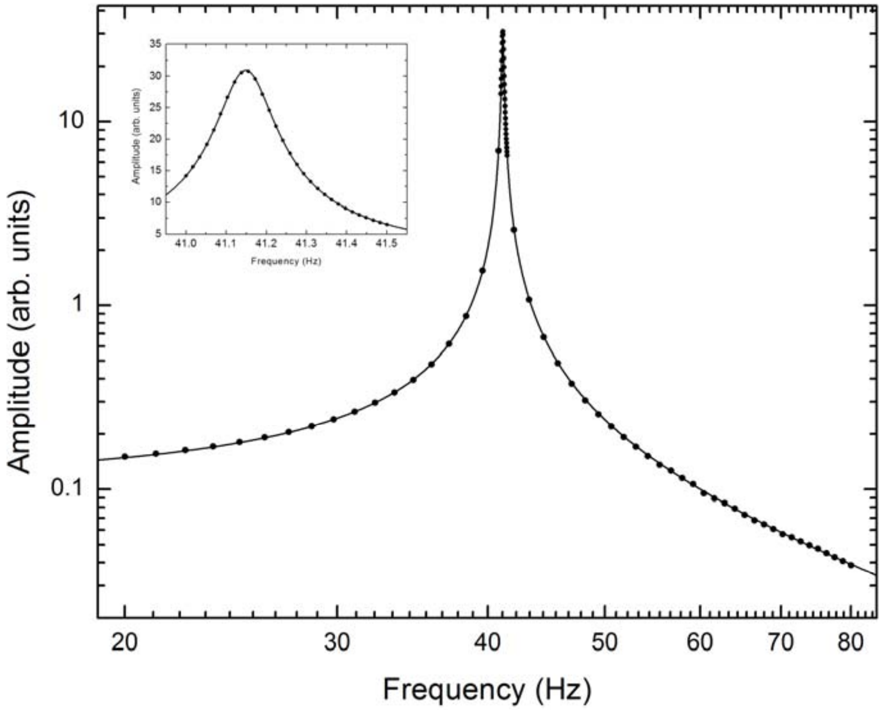

Figure 2. Another investigation of the driven harmonic oscillator, this time acquiring better data by computer, covering a larger frequency range. The inset plot shows a closer view of the same data points near the resonance peak, this time plotted on a linear scale. Again the data are well described by the fit line (the same line is plotted in both graphs), the functional form coming from Simple Harmonic Oscillator theory.

covering nearly three orders of magnitude in the response function.

Regardless of whether one acquires data by hand or by computer, the oscillator behaves like an ideal simple harmonic oscillator with high precision, without the need to explain additional nonlinearities in the oscillator.

Transient behavior

In the previous sections we ignored the transient behavior of the oscillator and focused on its steady-state response. The transient response is interesting in its own right, however, and it is easily observed using the MMHO.

If one starts with the oscillator at rest, and then turns on a sinusoidal drive signal that is 1-2 Hz away from the resonance, the oscillator responds with a pronounced beating between the drive frequency and the resonant frequency. The effect is especially strong with the eddy current damper removed, as this increases the settling time of the oscillator.

The oscillator transient response can be seen by simply observing the laser sweep, and watching the pronounced beating behavior for various drive signals can be quite educational. Both the beating and the eventual decay to the steady-state behavior can be observed. Even without taking quantitative data, these observations give one a qualitative feel for transient behaviors and the beating phenomenon.

Phase measurements

Simple Harmonic Oscillator theory also predicts a well-defined phase shift between the sinusoidal driving force and the oscillator response. In particular, the two signals are roughly in-phase when the drive frequency is much lower than resonance, 180 degrees out of phase high above resonance, and 90 degrees out of phase on resonance.

The phase shift can observed by sending both the drive signal and the photodiode signal to two channels of an oscilloscope (the drive signal also being sent to the Drive Coil IN port). Displaying both signals shows the phase shift directly, and the oscilloscope can be used to measure the phase shift between the two signals.

It is also instructive to display the two signals in XY mode on the oscilloscope, as this gives a different view of the phase shift. When the two signals are 90 degrees out of phase, the XY signal is an oval with no tilt. Even quite small tilts are readily observable in XY mode, so this method can be used to quickly determine the oscillator resonant frequency to quite high precision.

III. The resonant frequency

The oscillator resonant frequency ν_0 can be measured by a variety of techniques. They all yield similar results, but with varying levels of precision. Examining the different methods can be an interesting lesson in measurements and uncertainties.

A first method for determining the oscillator resonant frequency is by real-time observation of the oscillator amplitude when driven. On resonance, the amplitude is maximized. This method is rather slow and crude, yielding accuracies of 0.1 – 0.01 Hz, depending on how much care is exercised.

Fitting the oscillator response curve gives higher precision, but this requires post-processing of the data, so does not give immediate feedback to students in the lab.

Observing the phase shift in XY mode on the oscilloscope (see the discussion above) yields accuracies of 0.01 Hz or better, and it does so much faster than using real-time amplitude measurements.

The highest precision can be obtained with the oscillator in Clock Drive mode (see below), and an accuracy of 0.001 Hz can be obtained in just a few minutes. With this highest precision, one soon discovers that the oscillator resonant frequency is not constant. Simply breathing gently on one of the support wires (thus changing its temperature) yields a measurable frequency shift.

IV. Excitation and decay

One straightforward method for observing the exponential decay of the oscillator amplitude (when the drive is turned off) is by making a video of the laser streak on the ruler. A smart phone (or Ipod, for example) works well for making such a video, provided it is mounted stably, preferably using a tripod. The video can be analyzed by playing it back frame-by-frame (QuickTime has this capability, for example), and measuring the laser streak against the ruler markings.

Figure 3 shows data taken this way, the behavior being well described by an exponential decay curve. The time constant extracted from these data is then related to the frequency and Q measurements that were derived from the oscillator response curve.

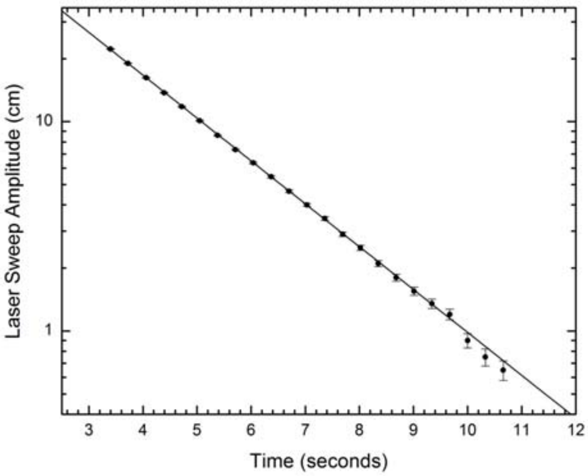

Figure 3. A measurement of the oscillator amplitude as a function of time, after the drive has been turned off. The data are well described by an exponential decay, as predicted by SHO theory. The decay time constant is related to the frequency and mechanical Q of the oscillator.

One can run the opposite experiment as well. With the drive frequency equal to the oscillator resonant frequency, start with the oscillator at rest and turn on the drive. The amplitude increases linearly at first, then leveling off to its steady-state value following a characteristic exponential function.

V. Clock Drive

High Q oscillators are the basis of most clocks, and the MMHO can be configured to run in this mode, thus providing an excellent introduction into the physics of clocks (horology).

In the MMHO internal electronics, the photodiode signal is used to generate a pulsed drive signal (see the *Instrument Description* document), which can be sent to the drive coil. This provides a positive feedback that makes the oscillator self-exciting.

Many students are surprised to discover that the oscillator in Clock Drive mode runs at its natural resonant frequency. For the MMHO, the two frequencies are equal to better than 0.1 percent.

One straightforward method to measure the resonant frequency in Clock Drive mode is to send the photodiode signal to the oscilloscope and use the frequency measure feature of the ‘scope.

A much more accurate method is to view the photodiode signal on the oscilloscope together with an independent square wave signal from the signal generator. Trigger on the sinusoidal photodiode signal and look for a drift in the position of the square wave on the ‘scope. Then adjust the frequency of the square wave until the drift is eliminated.

This drift method is extremely accurate because small phase shifts are readily observed on the oscilloscope. Moreover, the phase shift increases with time, so observing for a longer time yields a better frequency accuracy. Producing a drift of less than 0.05 radians in 50 seconds corresponds to a frequency accuracy of 0.001 Hz.

By measuring the oscillator in Clock Drive mode, students learn about high Q oscillators, feedback excitation, oscillator stability, and other aspects of the physics involved in making accurate clocks.

Quartz Crystal Oscillators

Using the MMHO in Clock Mode naturally leads one to a discussion of quartz crystal oscillators. Like the MMHO, these are mechanical clocks that use feedback to produce sustained oscillations. In the early days of electronic wristwatches, the quartz elements were shaped like tuning forks, running at precisely 32768 (2^{15}) Hz. A series of frequency dividers then yielded the one pulse per second signal desired to run a wristwatch.

After many decades of development, quartz oscillators are now typically much smaller, running at MHz frequencies with mechanical Qs in the millions, and they have become breathtakingly accurate as timepieces. Quartz oscillators are present in nearly all computers and cell phones, and are currently being manufactured at a rate of about two *billion* per year.

In another teaching moment, is it enlightening to point out that at the heart of the digital signal generator used with the MMHO lies a precision quartz mechanical clock, and this is what gives the signal generator its impressive frequency accuracy, adjustable down to the μHz level. The moral of the story is that mechanical oscillators, well described by SHO physics, play a central role in technology, whenever time or frequency precision is needed.

VI. Laser strobe feature

With the oscillator running in Clock Drive mode, the laser strobe feature in the MMHO can be used for visualizing the test mass oscillations. By strobing the laser on briefly each cycle, with the strobe frequency set 1 Hz away from the oscillation frequency, one observes a short laser streak on the ruler, moving back and forth at 1 Hz. By strobing at N times the oscillation frequency, one observes N short streaks. This is a simple feature, but it is always enjoyable for students who have not played with strobes before.

VII. Measuring the magnetic moment of the test mass

The MMHO oscillator parameters can be characterized via a number of routes. A measurement of the resonant frequency gives κ/I, where κ is the torsional spring constant and I is the mass moment of inertia of the test mass. The magnet moment of inertia can be calculated from its mass and physical dimensions; the remaining parts of the test mass are smaller and lighter, thus adding only about an additional 20 percent to I. The spring wire contribution to I is completely negligible.

With the spring constant κ known, the magnetic moment μ of the test mass can be determined by measuring the oscillator deflection resulting from a known magnetic field, the latter calculated from the geometry of the drive coil and the applied current.

Another method for determining μ is to apply a magnetic field using the bias coils. This field changes the spring constant of the oscillator, and thus the resonant frequency, and the latter can be accurately measured in Clock Mode. By measuring the frequency change as a function of the current in the bias coils, the magnetic moment of the test mass can be extracted. Because this method uses frequency changes to measure μ, it yields more accurate results than the straight deflection method.

VIII. Parametric Drive

The bias coils can also be used to drive the oscillator, using a method called *parametric drive*. Current through the drive coils changes the oscillator spring constant (one of the parameters of the oscillator), and this will pump up the oscillator amplitude when driven at *twice* the oscillator resonant frequency.

Below some threshold drive amplitude, the changing spring constant is insufficient to excite the oscillator appreciably. With the oscillator starting at rest, a small $2\nu_0$ signal sent to the bias coils initially does nothing.

Once the drive is increased above threshold, however, the oscillator amplitude increases exponentially with time. In practice, this response is quite dramatic to watch.

The physics behind parametric drive is somewhat subtle (see the *Magneto-Magnetic Harmonic Oscillator Physics* document), so this exercise is best appreciated by advanced students. It is instructive to show this to introductory students as well, however. The experiment itself takes little time, and it serves to show students what lies ahead in more advanced physics courses.