Newtonian Labs Teaching Guides
The Physics of Electrodynamic Ion Traps
A Qualitative Description

I. Introduction
The purpose of this Teaching Guide is to examine the physics behind electrodynamic ion traps, and specifically to take a close look at ion trapping in the Newtonian Labs Electrodynamic Ion Traps (EIT) experiment. Electrodynamic ion traps, also known as Paul traps, provide a means to guide the motion of charged particles, especially confining charged particles in free space using only time-varying electric fields. Wolfgang Paul and Hans Dehmelt received the Nobel Prize for Physics in 1989 for developing ion traps, and this technology is still in widespread use today.

Electrodynamic Paul traps operating at MHz frequencies can hold individual atomic or molecular ions, and these traps are commonly used as starting points for physics experiments investigating atomic or molecular properties. Paul traps have been used for building rudimentary quantum computers as well, in which the individual ions serve as qubits that can be examined using probe lasers. Ion trapping techniques are also commonly used in chemical mass spectroscopy, for general chemical analysis as well as for determining the molecular weights of complex compounds with high precision.

Our focus here will be on trapping larger particles -- about 25 μm in diameter -- using electric fields oscillating at 60 Hz. The ion traps themselves are a few centimeters in size, operating in air, and the particles are illuminated using laser light, making them easily visible to the naked eye. Additional information about the EIT apparatus can be found in our Instrument Description document. Various experiments that can be done using the instrument are described in our Guide to Experiments document. Both are available online at NewtonianLabs.com.

Earnshaw's Theorem
The basic idea of an electrodynamic ion trap is to confine a charged particle in free space, away from any other matter, using electric fields alone. There is a famous theorem, called Earnshaw's theorem, stating that one cannot construct a stable ion trap using electrostatic fields alone. To trap a positively charged particle at some position in space, for example, the electric field vectors around that position would all have to be pointing inward. And Maxwell's equations, specifically Gauss's Law, tell us that this is impossible unless there is a net negative charge at that position. So, try as one might, fundamental laws of physics tell us that it is not possible to create a static electric field geometry that will stably trap charged particles in free space.

There are magnetic variations of Earnshaw's theorem as well, for example stating that you cannot stably trap a bar magnet in free space using only static magnetic fields. Adding gravity does not help, and another version of Earnshaw's theorem states that you cannot levitate a stationary permanent magnet using only static magnetic fields.

Fortunately, there are many routes around Earnshaw's theorem. One popular engineering method is to use active feedback. In the magnetic case, for instance, one can continually measure the position of a levitated magnet and adjust the forces appropriately to keep the magnet positioned where you want it. This method is effective and relatively inexpensive, and magnetically levitated trains often use active feedback in their operation. Another way around the magnetic Earnshaw's theorem is to use a spinning magnet instead of a stationary one. There is a toy called the Levitron (easily found online) that demonstrates levitation of a spinning magnet without using active feedback.
Paul and Dehmelt got around Earnshaw's theorem by using oscillating electric fields, since the theorem strictly applies only to static fields. It is not immediately obvious that you can use oscillating fields to trap particles (hence the Nobel Prize), so our first task is to understand the basic physics behind all this.

II. Particle dynamics in oscillating electric fields

A uniform-field example

To begin, consider a charged particle placed between two conducting plates, as shown schematically in Figure 1 below. Assume that the plates are large compared to their separation, and that there is a vacuum between the plates. We then apply an oscillating voltage $V_{AC}$ to the plates, as shown in the Figure. As long as the fields do not change too rapidly, there will be a uniform electric field in the space between the plates, and the field will be proportional to the applied voltage.

Between these plates we place a charged particle at some position $z$, with zero initial velocity. The electric force on the particle is given by $F = qE$, and we will ignore the much weaker gravitational force. Since the electric field oscillates in time, so does the force on the particle. Thus the particle is pushed up and down as the force on it oscillates. But the force up equals the force down, so averaged over time there is no net force on the particle. Although the particle oscillates up and down, its average position $<z>$ does not change.

![Figure 1. A charged particle placed initially at rest between two conducting plates. An oscillating voltage $V_{AC}$ is applied to the plates, so the electric field oscillates with time. At any given time, however, the field is uniform between the plates. The oscillating electric field causes the particle position to oscillate. When the particle position $z$ is high (left), the electric field pushes it down. And when the particle $z$ is low (right), the electric field pushes it back up. The average particle position $<z>$ remains constant.](image-url)
Now note that the particle position is 180 degrees out of phase with the applied force, as is shown graphically in Figure 1. When the particle is up (relative to $<z>$), the force pushes it back down. And when the particle is down, the force pushes it back up again.

For another example, consider rolling a ball on a table back and forth between your left and right hands. When the ball is in your left hand, you push it to the right. And when the ball is in your right hand, you push it to the left. In both cases, we see that there is an out-of-phase force that maintains the oscillatory motion.

Adding an electric field gradient

Now we make the problem a bit more interesting by adding a field gradient, so the electric field is no longer uniform in space. One way to add an electric field gradient is to curve the two plates a bit, as shown in Figure 2 below. For example, the two plates in the Figure might be sections of spherical shells, where the geometrical centers are both located at the same point high above the plates.

The details of the plate geometries are not especially important. What is important is that the electric field lines look roughly like those shown in Figure 2 – in particular, the field strength near the top plate is higher than near the bottom plate (note the lengths of the field vectors in the Figure.) Since we only curved the plates slightly, we haven’t changed the field much, so the particle still behaves much as it did before – over timescales short compared to the oscillation period, the particle mostly just oscillates about its average position $<z>$.

But now we can see, just from the geometry in Figure 2, that the force averaged over one cycle is no longer zero. As shown in the Figure, when the particle is above its time-averaged position $<z>$ (left side of the figure), it experiences a stronger-than-average electric field pushing it downward. And when the particle is below $<z>$ (right side of the figure), the upward force is weaker than average. This imbalance was not present in Figure 1. From this fairly basic reasoning, shown graphically in Figure 2, we deduce that there is a net force pushing the particle down. So in this example the average position $<z>$ is not constant; instead $<z>$ changes on timescales long compared to the oscillation period.

In talking about ion trapping, we often separate the particle motion into short-term and long-term parts. On timescales short compared to the oscillation period, the particle motion is dominated by its simple back-and-forth motion. This motion is caused by the oscillating electric field at $<z>$, and is roughly independent of the field gradients. We call this the particle *micromotion*. This motion is fast, oscillatory, and it is caused by the local electric field.

Over longer timescales we talk about a *secular motion*, caused by the time-averaged secular forces.

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**Figure 2.** A charged particle placed initially at rest between two curved plates. The geometry of the plates causes a gradient in the electric field strength - the field is stronger for larger $z$ (as shown by the longer arrows). The imbalance means that the electric force on the particle is stronger at the top of its motion (left) and weaker at the bottom (right). Averaging over time, there is a net force that pushes the particle down, toward the weaker-field region.
The secular forces are generally weaker than the short-term oscillatory forces, and the secular motion is slow compared to the micromotion. The secular forces do depend on the overall electric field geometry, including the field gradients, and it is these forces can be used to trap charged particles.

An important point that comes out of this brief analysis is that the secular forces push particles toward regions of weaker oscillating electric fields. We demonstrated this for a special case above, but it turns out to be generally true over a broad range of conditions. For any physically reasonable oscillating field geometry, the secular forces always push particles toward regions of weaker oscillating fields.

III. Quadrupole ion traps

Now that we have a basic understanding of how particles behave in oscillating electric fields, we can proceed to make an ion trap by considering more complex field geometries. We will focus on what are called quadrupole ion traps.

One relatively easy way to make this type of trap is shown in Figure 3. An AC voltage is applied to two ball-shaped electrodes in a grounded box, creating oscillatory electric fields inside the box. If you are familiar with electric field lines around charged objects, then you may recognize that the electric field in the region between the two balls will look like what is drawn in Figure 3.

In particular, if you think about the exact spot halfway between the balls, you should soon deduce that the electric field must go to zero at that point. The reason is because of symmetry. The electric field from the top ball, for example, exactly cancels the electric field from the bottom ball. Everything is symmetrical around this center point, so the electric field must equal zero at that point.

Most of us recognize electric fields like these solely because of our experience in physics classes, and from seeing drawings and demonstrations of electric and magnetic field lines. This is not the kind of thing one normally has direct experience with. The mathematics behind the electric field geometry (Maxwell’s equations) is quite sophisticated, especially in full 3D, so showing exactly why the electric field looks like it does is not a simple problem. But our goal here is not prove every detail in the physics, but rather to provide a reasonable qualitative picture of ion trapping.

Another approach for making a 3D quadrupole trap is the Ring Trap shown in Figure 4. Again the electric field at the center of the ring is always zero, and the arrows show the field lines in the vicinity of this central region, at one point in the cycle.

If we do a much better job drawing the electric field lines between the two balls, or near the center of the Ring Trap (using some of that sophisticated

Figure 3. This sketch shows one method for making an ion trap, using two ball electrodes in a grounded box. When the balls are at a high potential relative to the box, the electric field lines between the balls are shown roughly by the arrows. A half-cycle later the balls will have a negative charge, and the field lines will be reversed. By symmetry, the electric field halfway between the balls is always zero.

Figure 4. This sketch shows another method for making an ion trap, using a ring electrode (seen edge-on here) inside a grounded box. The potential between the ring and the walls is set by an applied AC voltage. This geometry is used in the Ring Trap, one of three ion traps that are part of the EIT instrument.
mathematics), then we obtain the field geometry shown in Figure 5, which is called as a 3D quadrupole field. This Figure shows in detail how the electric field goes to zero at the center point, and the field increases in all directions outward from the zero-field point.

![Diagram of electric field geometry](image)

Figure 5. This sketch shows the electric field geometry near the center of the Ring Trap, as a function of radius \( r \) and height \( z \) relative to the center of the ring. Specifically, this shows the electric field vectors when the voltage on the ring is at its maximum. The length of each vector is proportional to the strength of the electric field at its midpoint. Multiply each vector by \( \cos(\omega t) \) to obtain the electric fields at other times. This type of “in from the sides, out the top and bottom, zero in the middle” electric field geometry is called a 3D quadrupole.

Now take a deep breath, and we can put it all together. Combine the 3D quadrupole field geometry (Figure 5) with the fact that the secular forces push charged particles toward regions of weaker electric field strength. Bingo -- the secular forces will push particles toward the zero-field point, trapping them there in free space.

At this point one really has to pause and digest for a minute, since the result is not immediately obvious or intuitive. Making some sketches of the electric field lines can be useful for visualizing the quadrupole field geometry, and one really needs to ponder the dynamics a bit to fully appreciate the secular forces described graphically in Figure 2. Although the underlying physics is essentially just basic mechanics, the result is subtle nevertheless, enough so to make it all quite interesting.

**Seeing is believing**

The Electrodynamic Ion Traps instrument is an excellent tool for teaching about this fascinating phenomenon. With it students can place particles into an ion trap, vary the trapping parameters, apply external forces, and observe the resulting particle dynamics with their own eyes. There is simply no better way to make the physics real than through personal experience.

Figures 6 and 7 show two examples of particles held in a Ring Trap. The ions exhibit micromotion oscillations 60 Hz, and these long-exposure photographs (longer than 1/60th of a second) show tracks that delineate the paths of each particle.

The particles all have like charges, so their mutual repulsion keeps them apart, while at the same time trapping forces push the particles toward the trap center. The resulting lattice-like structure is called a *Coulomb crystal*. The particles themselves are about 25 microns in diameter, and they are illuminated with green laser light, making them easily visible to the naked eye.

![Photograph of particles in Ring Trap](image)

Figure 6. A collection of particles held in the Ring Trap. The inner diameter of the ring is 16 mm.
Figure 7. Eight particles in the trap.

Figure 8 shows a composite photo of a single particle in the Ring Trap. In the photo on the left we can see how gravity pulls the particle downward until the trapping forces provide an equal upward force. This balance occurs when the average particle position $<z>$ is below the center of the trap. And at that position the particle exhibits some micromotion.

In the photo on the right, we added a static electric field, so that the qE force balances the gravitational force. The particle then behaves as if it were in a zero-gravity environment, so the trapping forces push the particle to the center of the trap, where there is no remaining micromotion.

With a single 25-micron-diameter particle, the gravitational and qE forces are some tens of picoNewtons. Although these are extremely small forces, their effects are easily seen in the EIT apparatus.

Figure 9 shows the trap with some hundreds of trapped ions, this time viewed from the plane of the ring. Note that the particle micromotion traces out the local electric field vectors, nicely demonstrating the quadrupolar field geometry.

Figure 8. (Left) A single trapped particle, displaced from the trap center by the static force of gravity. At this position it exhibits micromotion, so the photograph shows a vertical streak. (Right) An additional static electric field balances the gravitational force, so now trapping forces push the particle to the trap center, where there is no remaining micromotion.

Figure 9. A large cloud of ions held in the Ring Trap, viewed from the plane of the ring.